

# SPARSE ARRAY DESIGNS BASED ON ULA FITTING: PRINCIPLE AND APPLICATIONS

Yingsong Li, Wanlu Shi  
Anhui University, China; Harbin Engineering University, China



**Yingsong Li** is a full professor of Key Laboratory of Intelligent Computing and Signal Processing, Ministry of Education of Anhui University. His research focuses Signal/image processing, Array and antenna designs, MIMO radars, meta-surface designs for radar and communications and EMC.



**Wanlu Shi** is a Ph.D. student in Harbin Engineering University. Her research focuses on Sparse array design, DOA estimation and Beamforming.

## Introduction:

Sparse array has been considered for enhancing the uniform degree-of-freedom (DOF) and achieving high resolution in direction-of-arrival (DOA) estimation and beamforming. We proposed sparse array design method based on uniform linear arrays (ULA) to find out a good solution for aforementioned problems, giving a name of ULA fitting scheme for sparse array configurations. The proposed ULA fitting scheme and its variants are promoted for DOA estimation, target recognition and MIMO radar. The results obtained from the experiments and simulations verified that the proposed schemes outperform the traditional ULA and existing sparse arrays with low mutual coupling.

## 1. Principle of the proposed ULA fitting scheme

As we know, the traditional array signal processing [1] based on subspace technique like Multiple Signal Classification (MUSIC) [2] can detect less targets than the total number of sensors for DOA estimation, which also requires an inter-element spacing of maximum of half-wavelength to avoid aliasing. However, the mutual coupling will be high since the array element are setting very near. For the sparse array developments, our goal is to recognize more targets with less array elements in comparison with the traditional ULAs. Then, difference coarray (DCA) was reported to achieve high degree-of-freedom (DOF) [3]. Based on DCA concept, nested and coprime arrays [4-5] were presented for developing different sparse array structures, which can provide closed-form expression and good DCAs.

Although many sparse arrays have been proposed and discussed for many applications, there is no general method for sparse array constructions.

Our proposed method uses a polynomial model to analyze DCA and establish a relationship between positions of sensors and weight function based on a concatenation of a series of ULAs, which is denoted as ULA fitting scheme [6-9]. As a result, the proposed ULA fitting scheme can be used for constructing various sparse arrays with closed-form expression and DCA analysis. Additionally, a pseudo polynomial function was constructed to design high performance sparse arrays. In the ULA fitting scheme, the three different layers, namely the base layer (BL) that includes several sub-ULAs, the transform layer (TL) that has different forms and the additional layers (ALs) that are to complement weights. The base layer always has sub-ULA(s) that has same structure(s) to construct a dense coarray following the transform layer periodically, and the transform layer is to provide a transfer period including the transforming times, and additional layer together with base layer pad each period to get a tense coarray. The work principle of the sparse array (SA) design is illustrated in Fig.1. In this example, there are six sub-ULAs, where Sub-ULA3 is used as the transform layer, Sub-ULA1, Sub-ULA4, and Sub-ULA6 are adopted as the base layer, while Sub-ULA2, and Sub-ULA5 are used as the additional layer. Using the ULA fitting schemes in [6-9], the SA is constructed to obtain a

transform range in the DCA domain. Herein, the self-difference coarrays (SDCAs) of the transfer sub-ULA provides the periodic frame to be filled by the inter-difference coarrays (IDCAs) related to the transfer sub-ULA.

## 2. Example of an SA using ULA fitting

Based on the ULA fitting (UF) scheme, an example for SA design using 5 base layer (BL) is proposed and used for DOA estimation, which is denoted as UF-BL5. The UF-BL5 structure using the ULA fitting is presented in [10] to implement DOA estimation, and the obtained performance compared with existing SAs is presented in Fig. 2.

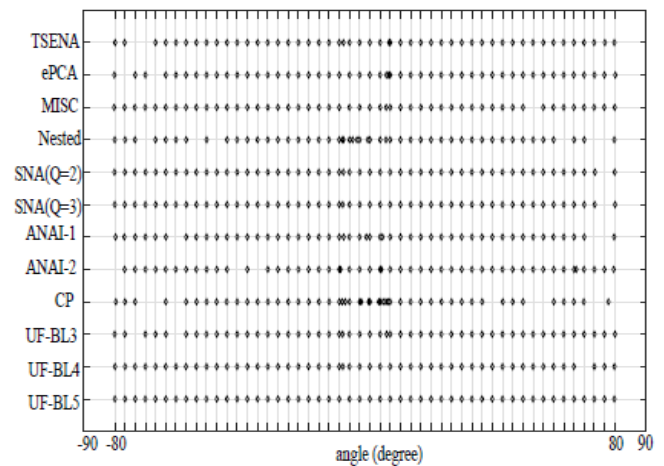


Fig. 2: DOA estimation with different SAs [10]

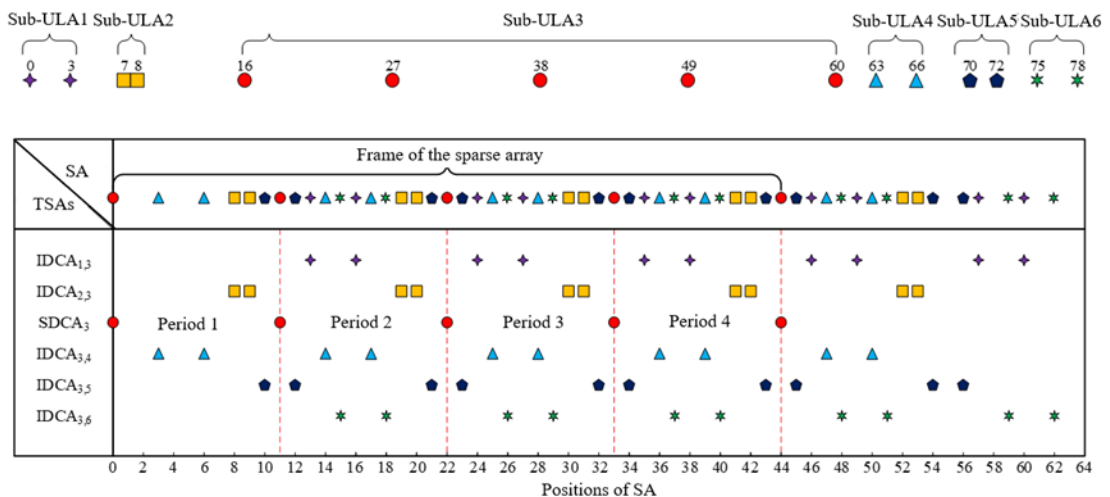


Fig.1 Illustration for SA design using ULA fitting scheme

Herein, UF-BL3 and UF-BL4 are SAs using ULA fitting with 3 and 4 BLs, respectively, and their performance is compared with Two side extended nested array (TESNA) [11], Extended padded coprime array (ePCA) [12], Maximum inter-element spacing constraint (MISC) [13], Nested arrays (Nested) [4], Super nested array (SNA) [14], Augmented nested arrays (ANA-1 and ANA-2) [15], Co-prime arrays (CP) [5].

### 3. Data processing for DOA estimation in practical engineering

To promote the application of the SA design using ULA fitting and discuss its performance, an experiment is carried out in the South China Sea. The receiving array has 80 uniformly located hydrophones, where the interspace is 6.25 meters and the deployment depth is 120 meters. The observed space is  $[-90^\circ, 90^\circ]$  and the direction of the receiving ship (fixed above the receiving array) is  $65^\circ$ , while some other ships are working around. The analyzed frequency ranges from 50 to 120Hz with an observation duration of 500 seconds. In the data analysis, only 15 hydrophones are used for the SAs. The performance for various sparse arrays for analyzing the obtained data is shown in Fig.3.

From the results, we can see that the proposed adjoint transfer layer (ATL) with 1 BL and 2BL are denoted as ATLI-1BL and ATLI-2BL that have better resolution and tracking performance than most existing SAs.

### 4. UF scheme for MIMO radar

Also, to make the sparse array suitable for practical applications, the ULA fitting scheme is used for developing MIMO radar with high uniform DOF and low mutual coupling. In the design, the transmit array has a DCA with holes is considered. The devised sparse MIMO arrays maintain the merit of UF principle and they have closed-form expressions. Herein, an improved ULA fitting scheme proposed in [17] and an SA with high uniform DOF is designed accordingly using the adjoint transfer layer with an increasing number of sensors and 1-base layer (ATLI-1BL). For the MIMO radar, an SA developed using ULA fitting having 3 BLs, which is mentioned as UF-3BL is selected as the transmit array and ATLI-1BL is chosen as the receive array. In this case, the devised MIMO radar is represented as UF3-ATLI1BL MIMO, where 11 transmitters and 7 receivers are used for the MIMO radar. Also, we have a unit element spacing for the phototype array (PA). The design procedure of the UF3-ATLI1BL MIMO radar is presented in Fig.4.

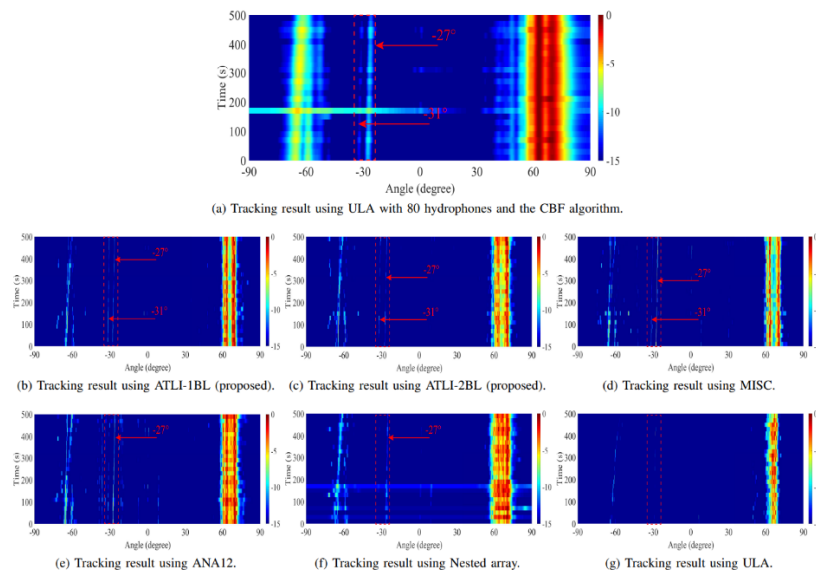
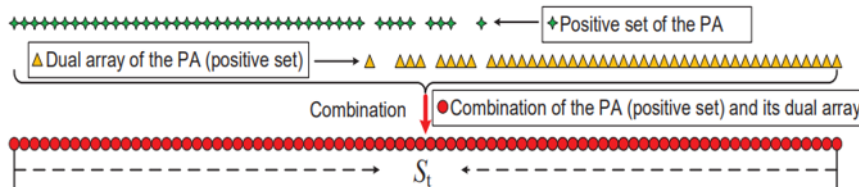


Fig.3 Comparisons of tracking results with different sparse arrays [9]



(a) UF3-ATLI1BL MIMO with  $M_t = 11$ ,  $M_r = 7$ , and  $S_p = 1$ .



(b) Processing of determining  $S_t$  for the UF3-ATLI1BL MIMO [7].

Fig. 4 Illustration of the UF3-ATLI1BL MIMO (with 18 physical sensors) and the design process

### 5. Conclusion

ULA fitting sparse array is a more general method for developing sparse arrays to achieve high degree-of-freedom and low coupling leakage. Form the investigation, the ULA fitting scheme can be used for MIMO radar, DOA. In the future, the ULA fitting will be used for beamforming and 2-D applications.

#### REFERENCES

[1] P. K. Bailleul, "A new era in elemental digital beamforming for spaceborne communications phased arrays," Proc. IEEE, vol. 104, no. 3, pp. 623–632, Mar. 2016.

[2] R. Schmidt, "Multiple emitter location and signal parameter estimation," IEEE Trans. Antennas Propag., vol. 34, no. 3, pp. 276–280, Mar. 1986.

[3] R. T. Hoctor and S. A. Kassam, "The unifying role of the coarray in aperture synthesis for coherent and incoherent imaging," Proc. IEEE, vol. 78, no. 4, pp. 735–752, Apr. 1990.

[4] P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," IEEE Trans. Signal Process., vol. 58, no. 8, pp. 4167–4181, Aug. 2010.

[5] P. P. Vaidyanathan and P. Pal, "Sparse sensing with coprime arrays," in Proc. Asilomar Conf. Signals, Syst., Comput., Pacific Grove, CA, USA, Nov. 2010, pp. 1405–1409.

[6] W. Shi, S. A. Vorobyov, Y. Li, ULA Fitting for Sparse Array Design, IEEE Transactions on Signal Processing, vol.69, pp.6431-6447, 2021.

[7] W. Shi, X. Liu, Y. Li, ULA Fitting for MIMO Radar, IEEE Communications Letters, vol.26, 2022.

[8] W. Shi, Y. Li, S. Vorobyov, Low mutual coupling sparse array design using ULA fitting, IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2021), 6-11st, June, Toronto, Canada, 2021.

[9] W. Shi, Y. Li, Improved uniform linear array fitting scheme with increased lower bound on uniform degrees of freedom for DOA estimation, IEEE Transactions on Instrumentation and Measurement, 2022.

[10] Y. Yao, W. Shi, Y. Li, et al, Investigation on ULA fitting promoting low coupling sparse arrays, 2022 International Applied Computational Electromagnetics Society (ACES-China) Symposium, Xuzhou, 28-31 July, China, 2022.

[11] S. Ren, W. Dong, X. Li, W. Wang, and X. Li, "Extended nested arrays for consecutive virtual aperture enhancement," IEEE Signal Process. Lett., vol. 27, pp. 575–579, Mar. 2020.

[12] W. Zheng, X. Zhang, Y. Wang, J. Shen, and B. Champagne, "Padded coprime arrays for improved DOA estimation: Exploiting hole representation and filling strategies," IEEE Trans. Signal Process., vol. 68, pp. 4597–4611, Jul. 2020.

[13] Z. Zheng, W. Q. Wang, Y. Kong, and Y. D. Zhang, "MISC array: A new sparse array design achieving increased degrees of freedom and reduced mutual coupling effect," IEEE Trans. Signal Process., vol. 67, no. 7, pp. 1728–1741, Apr. 2019.

[14] C. L. Liu and P. P. Vaidyanathan, "Super nested arrays: Sparse arrays with less mutual coupling than nested arrays," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process., Shanghai, China, Mar. 2016, pp. 2976–2980.

[15] J. Liu, Y. Zhang, Y. Lu, S. Ren, and S. Cao, "Augmented nested arrays with enhanced DOF and reduced mutual coupling," IEEE Trans. Signal Process., vol. 65, no. 21, pp. 5549–5563, Nov. 2017.